

Kepler's Third Law

Key Stage 4

Topics covered: Planetary orbits, Solar System, graphs, astronomical units

Teacher's Notes

This extension activity illustrates Kepler's Third Law for our own Solar System using graphing methods, with a further look at how the same law can be used to derive information about the orbits of exoplanets around their parent stars.

Equipment:

Graph paper, pencil, and ruler for every student, and a student sheet with tables of data for our solar system and exoplanets orbiting other stars (page 5).

Class discussion before the activity:

Was Kepler the first to consider a heliocentric (Sun centered) model for the Solar System?

Answer: No. The Greek astronomer and mathematician Aristarchus (c.320–c.250 BC) was the first to consider a heliocentric model. It took roughly another 1800 years for the model to resurface with the work of Nicolaus Copernicus.

The word 'planet' comes from the Greek for 'wanderer'. Why do you think this is?

Answer: While the background stars in the night sky do not appear to move relative to one another, the planets do change in their position over time, relative to background stars.

Demonstrate circular motion with an object on a piece of string. Inform students that your fist holding the string is the Sun and the object is a planet. Swing it around. Now shorten the length of string and swing the object around again.

What two properties of the circular motion of the planet have changed and in what way?

Answer: The average distance between the Sun (fist) and the planet (object) is smaller and the period or the time it takes for the planet to return to the same position is shorter.

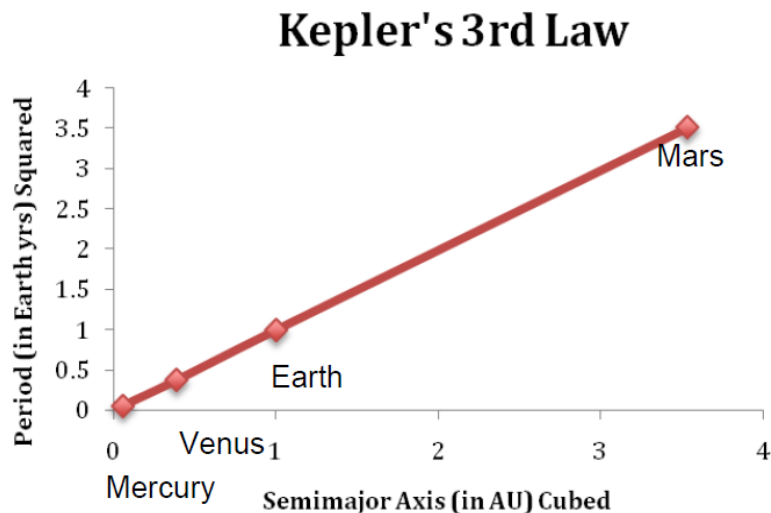
Background science

The questions posed to students above aim to prepare them to study Kepler's Third Law of planetary motion:

$$T^2 \propto r^3$$

Where T is the period of the planet and r is the average distance between the planet and its parent star.

By assuming a heliocentric model for our solar system and using 20 years of detailed observations by Tycho Brahe, Kepler empirically derived his three laws for the mechanics of orbits. He would have been unaware of the theory of gravity later introduced by Newton and instead theorised a magnetic force that drove the planets around their orbits. Although false, this was the first real attempt to explain the physics behind the motion of the planets and he correctly chose ellipses as the shape of planetary orbits as opposed to perfect circles.



Incorporating insights from Newton's theory of gravitation, Kepler's Third Law can be re-written in this more general form:

$$T^2 M_* = r^3$$

Where M_* is the mass of the star that the planet orbits in solar masses (i.e. $1 M_* =$ the mass of our Sun), T is the orbital period of the planet expressed in Earth years and r is the average distance between planet and star in Astronomical Units or AU ($1 \text{ AU} =$ the distance from the Earth to the Sun).

Over the last 15 years, observations of distant stars have shown that, just as our star, the Sun, supports a planetary system, so too, do many others. Newton's version of Kepler's Law can be used to learn more about these exoplanet systems from observations of these systems.

Stellar mass: Using spectrometers to study dispersed stellar light, astronomers are able to determine the temperature, size and mass of distant stars. This determination rests upon extensive study of starlight which has led to a classification system for stellar spectra and an understanding of how stars form and evolve.

Planetary period: The various methods of searching for exoplanets detect periodic wiggles in the parent star's position (radial velocity and/or astrometry method) or periodic dimming of the parent star's brightness (transit method), and yield an estimate of the period of the planet as it orbits its star.

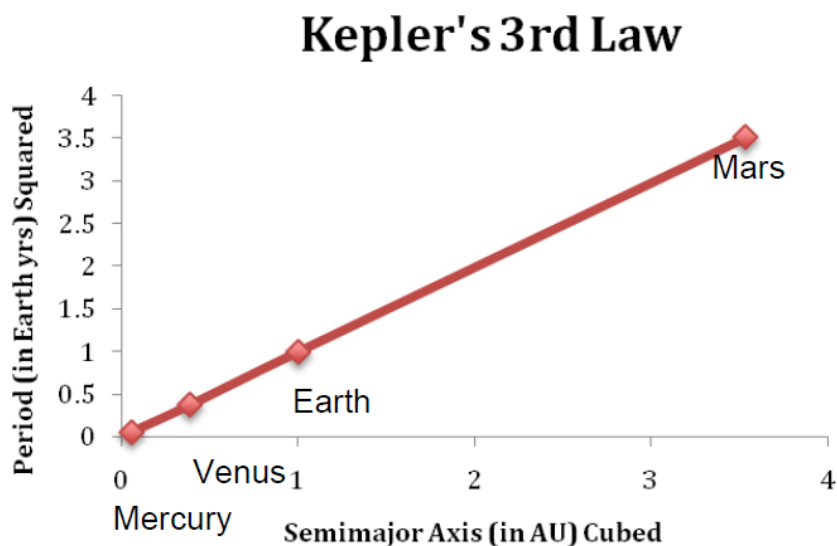
By inserting these two values into the equation above, astronomers are able to solve for the average distance of the exoplanet from the star.

Answers for Activity 1:

Proving Kepler's Law within Solar System

The first table in the student sheet at the end of this document gives the orbital period and average orbital distance for all the planets in our solar system. Students must square the period for each (T^2) and cube the average distance to the Sun (r^3). Students can then be asked to plot one against the other on their graph paper.

Combining the results for all the planets may be difficult considering the distances involved. An easier option is to just graph the terrestrial planets Mercury, Venus, Earth and Mars. Another graph may be attempted for the gas giants to complete the proof for our solar system.



Answers for Activity 2:

Distance of exoplanet from its star using Kepler's Third Law

The second table at the end of this document gives the orbital period for some of the exoplanets discovered by the NASA probe Kepler, which uses the transit method of detection. Ask students to work out the average orbital distance for each planet. Compare these with the distances of the planets in our solar system and ask students why these planets have little to no chance of supporting life forms. Remember that the distances will be in Astronomical Units (AU). The conversion for AU to km is given on the page of tables.

Students will need to rearrange the latter version of Kepler's Third Law and solve for r . They will then have the average orbital distance of each exoplanet. As can be seen from the results, many exoplanets that have been detected are orbiting very close to the parent stars with many completing one orbit in less than a week!

Activity: Kepler's Third Law

$$T^2 \propto r^3$$

Orbital Period squared is directly proportional to the average orbital distance cubed.

Orbital periods and distances for the planets in our Solar system

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Average distance to Sun (AU)	0.39	0.72	1	1.52	5.2	9.54	19.18	30.06
Orbital Period (Earth years)	0.24	0.62	1	1.88	11.86	29.46	84.01	164.8
Period squared								
Average distance cubed								

AU = Astronomical Unit

1 AU = Distance from Earth to Sun = 149,600,000km

$$T^2 M_* = r^3$$

M_* is the mass of the star that the planet orbits in Solar Masses (see below)

Orbital periods and stellar masses for a selection of exoplanets

	Kepler 4b	Kepler 5b	Kepler 7b	Kepler 8b	Kepler 9c
T (in Earth Years)	0.009	0.01	0.013	0.01	0.11
M_* (in Solar Masses)	1.2	1.4	1.3	1.2	1
r (in AU)					

1 Solar Mass = mass of the Sun = 2×10^{30} kg

Rearrange the last equation above and solve for r using the values in the table to find out how far away on average each exoplanet is from its star (in AU).