Calculating where space begins

Post-16

Topics covered: forces, gravity, lift, circular motion, orbits, atmosphere

Teacher's Notes

This activity will apply the concepts of gravity, orbits and aerodynamics to a practical problem. Using real world atmospheric data and technical specifications, students will be able to use Newton's law of gravitation and the definition of circular motion, along with a new concept, lift, to determine the altitude of the beginning of space. Finally, a mismatch between the accepted value and their calculated value will get students to think critically about the definition and why debate exists over the true edge of space. Answers to the questions are provided at the end of this guide.

This activity is a great accompaniment to the Royal Observatory Greenwich short animated video "Where Does Space Begin?" available on our website (https://www.rmg.co.uk/discover/teacher-resources/where-does-space-begin) or on our Vimeo page (https://vimeo.com/royalobservatory) along with many other videos produced by our team.

Equipment: calculator

Questions to ask the class before the activity:

How do planes fly?

Answer: Although the full answer is very complicated, the simple explanation is that the design of the plane's wing, along with its forward motion through air, produces a lift force upwards that counters gravity.

Could a plane fly in space in the same way as it does in the air?
Answer: No, without a lift force that comes from the atmosphere, the plane would not be able to fly the same way it does in the air.

How do satellites move through space?
Answer: Satellites are held in orbit by gravity, but must travel horizontally very fast to avoid falling back to Earth. These orbits are elliptical or circular. They do not require a lift force to remain at altitude and once moving fast enough, they rarely need to power their engines again as they have no atmosphere to slow them down.

Could a satellite orbit the Earth low down in the Earth's atmosphere?
Answer: No, the drag force from the atmosphere would rapidly slow the satellite and it would crash to Earth.

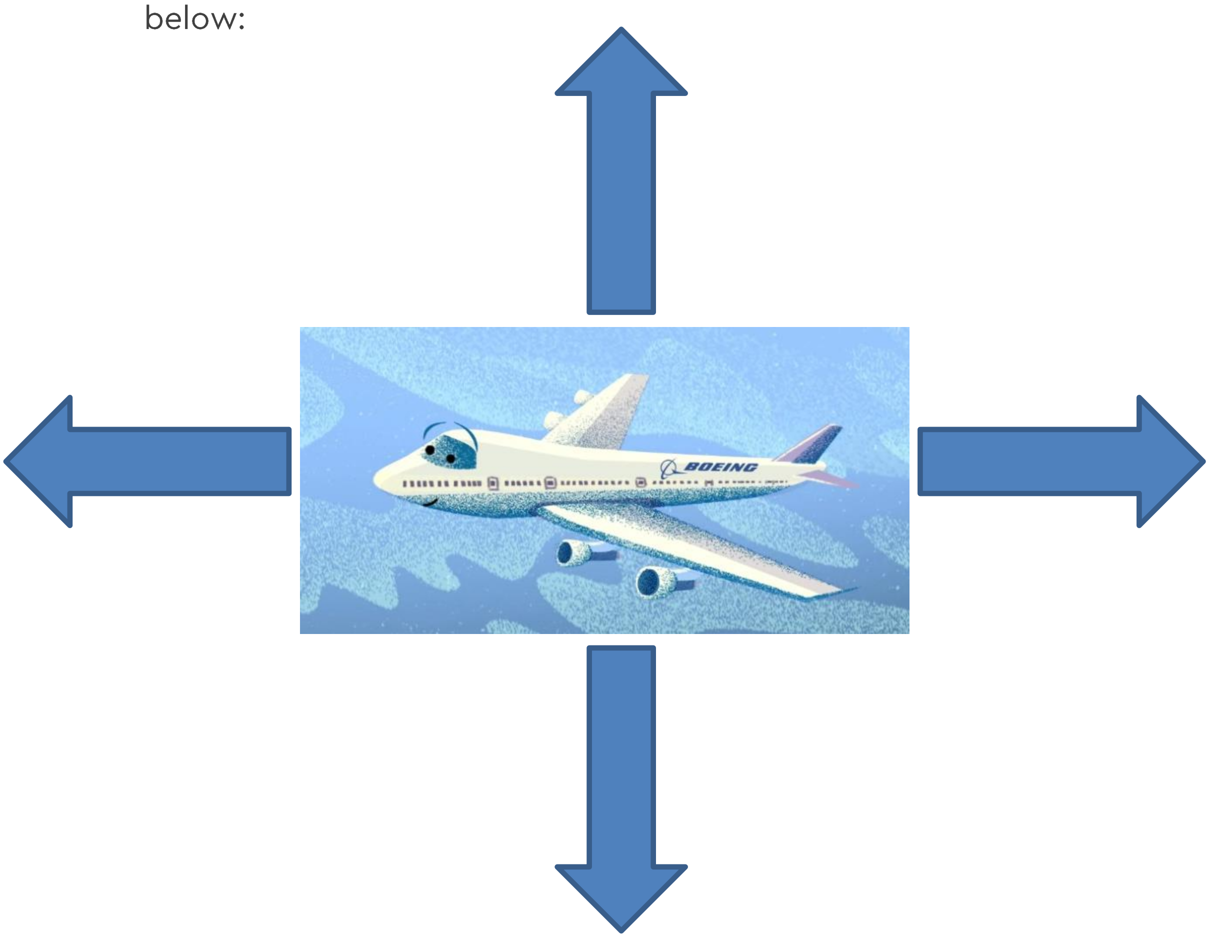
Questions to ask the class after the activity:

Would a plane/satellite flying at the altitude of the Kármán line be able to stay up in space permanently?
Answer: No. While it would be highly impractical to try, the atmosphere is technically capable of supporting flight well above the Karman line. This means that it does not disappear above the Kármán line, it merely becomes very thin. As such there is still a drag force on the object and it would quite quickly slow down and fall back to Earth. Even the International Space Station at 400km above the Earth's surface has to occasionally push itself back into its orbit due to the small effect of atmospheric drag. Instead the Kármán line represents where the speed required to provide enough lift for a plane is also the speed an unpowered satellite would need to orbit at, marking the approximate cross-over from aeronautics (travel within the atmosphere) to astronautics (travel within space).

Activity: Calculating the position of the Kármán Line

Ever wondered where space begins? Most scientists, astronauts and aeronautical engineers around the world agree it is at a place known as the Kármán line. Theodore von Kármán calculated this as the altitude at which regular planes would no longer be able to rely on lift to support their flight, instead travelling at speeds that are comparable to those needed to orbit. You are going to calculate the altitude of this line for a Boeing 747 aircraft.

1) As a plane travels through our atmosphere, there are 4 major forces acting upon it. Label the forces on the diagram using the words below:



GRAVITY

LIFT

THRUST

DRAG

2) For a plane in level flight (staying at the same altitude), what must be true about the size of the force of gravity and the lift force?

3) The force of gravity F_{grav} acting on a plane with mass m at an altitude h above the Earth (radius R_E, mass M_E) is given by the following equation:

$$F_{grav} = \frac{GM_Em}{(R_E + h)^2}$$

where G is the universal gravitational constant.

For a plane in flight, the lift force F_{lift} is based on the speed of the plane v_l and its surface area A and is given by:

$$F_{lift} = \frac{1}{2}\rho {v_l}^2 AC$$

where ρ is the density of air around the plane and C is a constant that is based on how aerodynamic the plane is.

a) Combine and rearrange these two equations so that the speed of the plane is the subject of the equation.

b) Calculate the velocity required for a plane to travel horizontally at an altitude of **1km** above the ground. **Hint: Be careful of your units!** Note that:

$M_E = 6 \times 10^{24}$ kg
$R_E = 6400$ km
$m = 333000$ kg
$\rho = 1$ kg m^{-3}

$G = 6.67 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}
$A = 500$ m^2
$C = 0.5$

As you move up through the atmosphere, the air becomes thinner with a much lower density.

c) Calculate the velocity required for a plane to travel horizontally at an altitude of **10km**, where the air has thinned to $\rho = 0.4$ kg m^{-3}

4) Satellites in orbit around the Earth rely on the force of gravity to keep them in motion. Assuming that they follow circular paths, the force that keeps the satellite in motion is:

$$F_{circ} = \frac{m{v_0}^2}{R_E + h}$$

where v_0 is the satellite's velocity.

a) Combine the circular motion equation above the force of gravity equation from 3) and rearrange to make the velocity of the satellite the subject of the equation.

b) Calculate the velocity a satellite would have if it were orbiting the Earth at the following altitudes:

Altitude (km)	Velocity (ms^{-1})
20	
40	
60	
80	
100	
120	
140	

c) What do you notice about these values? Why do you think that is? (Hint: think about the altitudes we are using here)

The Kármán line is defined as the altitude where the velocity needed to sustain lift in flight is also the velocity a satellite would orbit at. The velocity required to sustain lift at different altitudes is plotted in the graph below, based on real world atmospheric data. Be careful when reading the axes of this graph. The y-axis is plotted so that the labels jump by a factor of 10 each time.

5)

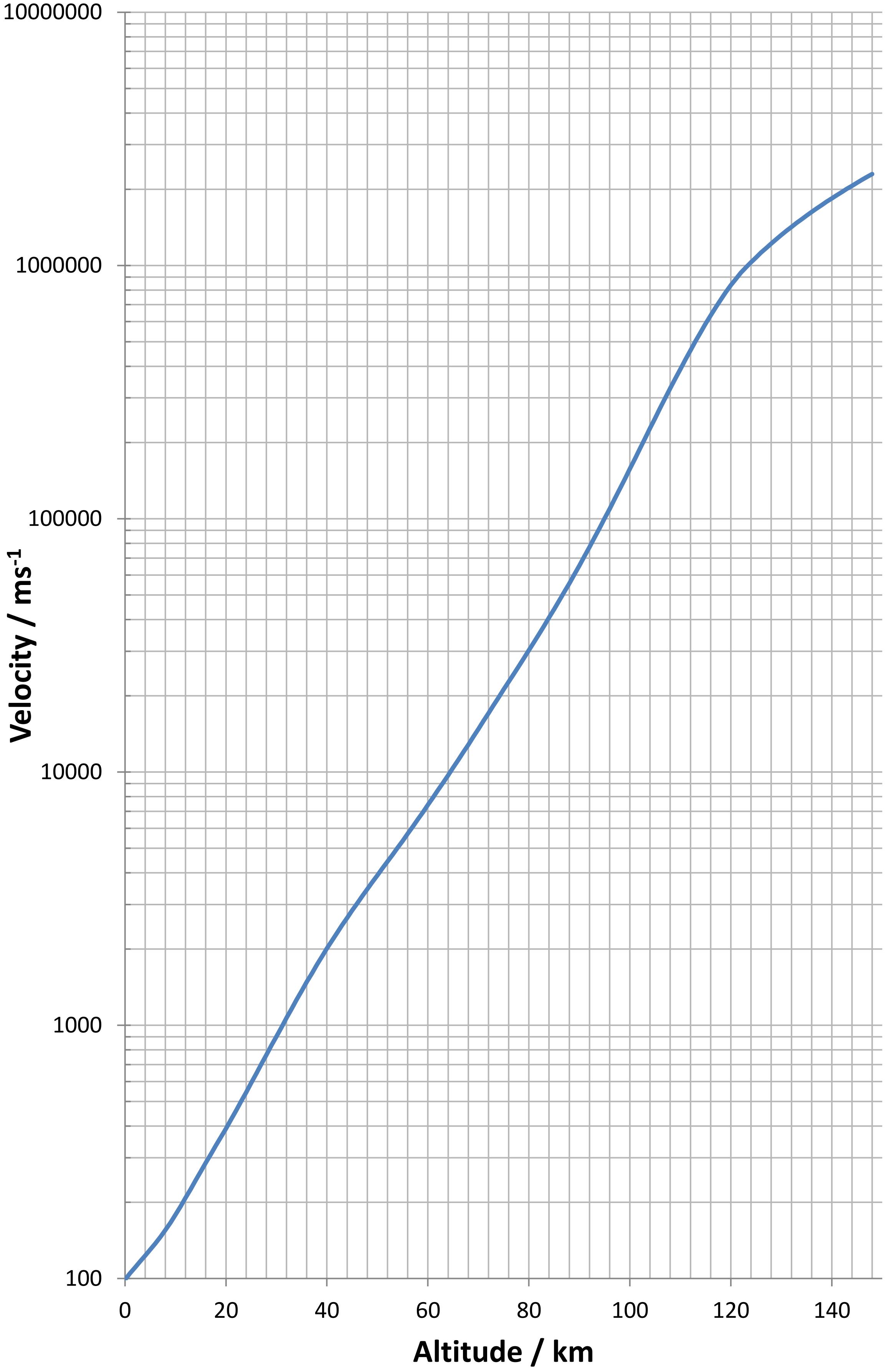
a) Complete the graph by filling in the data points from the table above.

b) Use the graph to determine the altitude of the Kármán line for the Boeing 747.

The Kármán line is usually defined as being at 100km above sea level. However there is some debate over exactly where the line should be and over whether a single definition is reasonable.

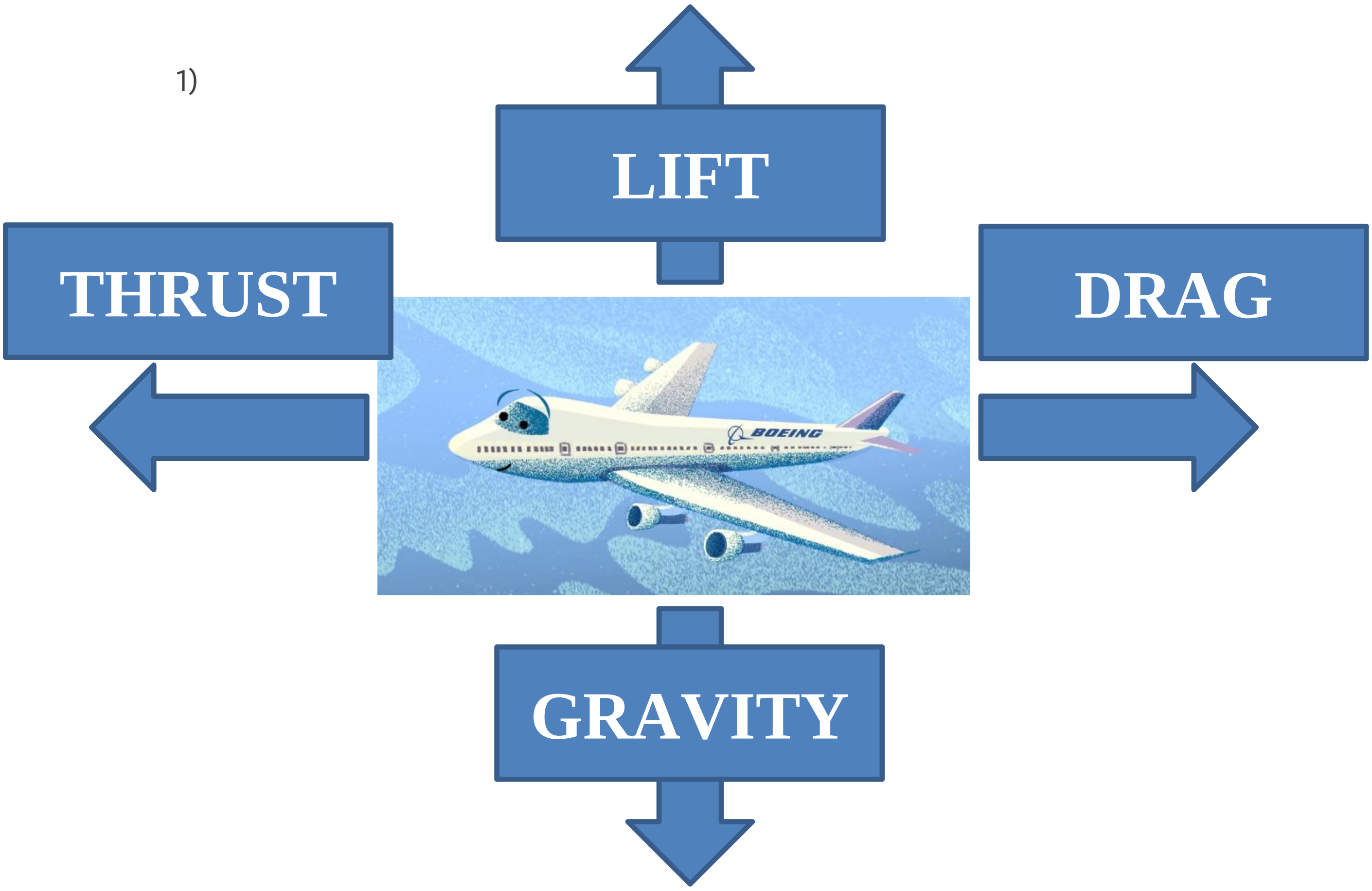
c) Why do you think such debate exists? Which variables might change under certain situations?

d) How does your value for the Kármán line compare with the usual value? What do you think is the reason for the difference?


10000000
1000000
100000
10000
1000
100
Velocity / ms-1
0
20
40
60
80
100
120
140
Altitude / km

Answers

1)



2) The forces of gravity and lift must be equal for the plane not to be accelerating upwards or different.

3)

a) $v_l = \sqrt{\frac{2GM_E m}{(R_E+h)^2 \rho A C}}$

b) For $h = 1$ km,
$v_l = 161$ ms^{-1}

c) For $h = 10$ km,
$v_l = 255$ ms^{-1}

4)

a) $v_o = \sqrt{\frac{GM}{R_E+h}}$

b)

Altitude (km)	Velocity (ms^{-1})
20	7895
40	7883
60	7871
80	7859
100	7847
120	7835
140	7823

c) There are multiple possibilities for what students may notice:
   i) The speeds are decreasing as the satellite moves away from the ground. The force of gravity is reducing so from the definition of F_{circ} the velocity of the satellite does not need to be so high to maintain a stable orbit.
   ii) The speeds are all very similar to one another. The altitudes used here are tiny compared to the radius of the Earth, meaning the changes between different altitudes are small.
   iii) The velocities are very high. These are all quite close to the ground (e.g. the International Space Station orbits at 400 km) so the velocities needed are extremely high.
   iv) A satellite would not be able to orbit at these altitudes. These are all well within the atmosphere, so atmospheric drag would be quite high. A satellite would only manage a few decaying orbits before it fell back to Earth.

5)

a) Graph is plotted below – plotted values are effectively a straight horizontal line at 8000 ms^{-1}

b) ~62km

c) The debate exists because the Kármán line is not a universal boundary and yet only a single definition is used. Different planes will have need to travel at different speeds in order to maintain lift so the altitude where an orbiting object's speed matches the plane's will depend on the plane. Also the atmosphere itself can change. Factors that impact the position of the Kármán line include:
   - the properties of the plane
     1. Mass – a heavier plane requires more lift and so a higher speed at the same altitude
     2. Surface Area – a smaller surface area provides less lift and so a higher speed is required to maintain lift

3. Aerodynamic constant – some planes are more aerodynamic than others. If a plane cuts through the air better, it will require less speed to maintain its altitude

- The properties of the atmosphere. This changes on short timescales (weather), on medium timescales (day and night cycle), on long timescales (seasons) and very long timescales (climate change), and it changes based on position (local weather and latitude and longitude).

d) The position of the Kármán line for a Boeing 747 is much lower in the atmosphere than the typical value. This is because the Boeing 747 is a very large, heavy and poorly aerodynamic plane.